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ANALYSIS OF A MOLECULAR BEAM SYSTEM UTILIZING A KNOWN PRESSURE SOURCE

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SUMMARY

An analysis is presented of a molecular beam system which utilizes the combined techniques of pressure attenuation, molecular beaming, and cryopumping to produce accurately known pressures in the ultra-high vacuum region. The system is shown to have a known molecular density with a maximum uncertainty of 6 percent over a variable range of pressures from 6.65×10^{-11} to 1.33×10^{-5} newton/meter^2 (5×10^{-13} to 10^{-7} torr). The kinetic behavior of gas flow in each component of the system is analyzed to determine the important system parameters and to establish their optimum values so that a maximum range of beam density is obtained.

INTRODUCTION

Vacuum instrumentation research requires an accurate knowledge of molecular densities for pressure measurements below 10^{-9} newton/meter^2. Heretofore molecular density determinations have been made indirectly from data obtained by pressure-indicating devices such as hot-filament and cold-cathode ionization gages. These devices indicate a pressure that is proportional to the molecular density of a given gas; but due to various inherent limitations, these gages are not adequate over the present working pressure range. For example, one problem in the hot-filament gage is the presence of residual X-ray currents below 10^{-8} newton/meter^2. In the cold-cathode gage, changes in sensitivity below 10^{-8} newton/meter^2 have been noted, along with intermittent extinguishing of the discharge. The development of an ultra-high vacuum-gage calibration apparatus, where gas number densities may be controlled and accurately determined, makes it possible to obtain known pressures from the direct calculation of the molecular density in a test chamber. An established means of number density calculation requires the calibration gas to enter a test chamber as a molecular flux. The approach used herein for establishing such a molecular flux is the molecular beam technique.

This paper describes a system incorporating the combined techniques of pressure attenuation, cryopumping, and molecular beaming. A detailed discussion of the number density calculation for an ideal gas under molecular flow conditions is also given.

Simultaneously with the work discussed in this paper, research of a similar nature was performed as part of Contracts NAS 1-2691, NAS 1-5347, NAS 5-3967, and NAS 5-9348. The work done under Contracts NAS 5-3967 and NAS 5-9348 is presented in reference 1. However, to gain a clearer understanding of the total work involved and of the independent study conducted at the Langley Research Center, it is pointed out that the present paper contains additional information such as experimental data obtained from a prototype system and the evaluation of probable errors associated with pressure measurements.

SYMBOLS

A	area, centimeters^2
C	conductance, liters/second
f(v)	Maxwellian distribution function for molecular speeds
k	Boltzmann's constant
L	length of gage ionization region, centimeters
l	distance from beam aperture, centimeters
M	mass of test fluid, grams
m	mass per molecule, grams/molecule
$\dot{N}$	molecular flux, molecules/second
n	molecular density, molecules/centimeter^3
p	pressure, newtons/meter^2 (133 newtons/meter^2 = 1 torr)
Δp	uncertainty in pressure measurement, newtons/meter^2
Q	gas flow rate, newton-liters/meter^2-second
r	radius, centimeters
S	pumping speed, liters/second
T	absolute temperature, ^{o}Kelvin

t	time, seconds
V	volume, centimeters^3
v	random molecular speed in furnace, centimeters/second
$\bar{v}$	average molecular speed, centimeters/second
Δx	uncertainty in parameter measurement
$Z = \left(l^2 + r_2^2\right)^{1/2}$	
Γ	molecular flux per unit area, molecules/centimeter^2-second
δ	thickness, centimeters
θ	any angle, degrees
ρ	density of test fluid, grams/centimeter^3
ϕ	angle from aperture normal (may be as large as 360^o), degrees
ω	solid angle, steradians

Subscripts:

a	molecular beam aperture
b	molecular beam
d	detector
e	gage enclosure aperture
f	molecular furnace
g	ionization gage
o	source at time zero

p	porous plug
t	source at time t
v	function of velocity
τ	source at equilibrium
1,2	beam aperture and enclosure aperture, respectively (see fig. 3)

MOLECULAR BEAM SYSTEM

General Description

A molecular beam may be considered to be a stream of molecules effusing from a small source into an evacuated chamber where its direction is defined by collimating apertures. The system described in this paper consists of a high-pressure (13.3 to 1.33×10^6 newtons/meter^2) gas source which supplies gas through a porous plug to a molecular furnace. From the molecular furnace, a beam is extracted and injected into a cryopumped gage calibration cavity.

The system is shown schematically in figure 1. The extreme-high vacuum section consists of a vacuum enclosure (guard vacuum), liquid nitrogen auxiliary cryopump, and liquid helium cryopump which contains the molecular beam core. In the typical design of extreme-high vacuum systems, the guard vacuum, produced by a combination of diffusion and mechanical pumps, provides some degree of isolation for the 77° K liquid nitrogen cooled wall and reduces the consequence of leaks. The purpose of the liquid nitrogen wall is to reduce the dissipation of heat radiation incident on the 4.2° K liquid helium cooled inner wall. Cryopumping is accomplished at the surface of the cryogenic walls by a molecular adsorption process. Effective cryopumping reduces the background pressure so that it becomes negligible in calculating the beam density and eliminates backscattering of the beam molecules. A diffusion-pump and mechanical-pump combination is utilized with liquid nitrogen baffles at the extreme end of the molecular beam to pump away noncondensable gases that may exist in the system. A demountable gage support is used to change the location of the gage at 10 and 117 centimeters along the axis of the beam. Two gage position mountings are required to cover the necessary beam densities; however, a shutdown of the system is required to change the gage location.

System Components

Gas source.- The high-pressure gas source (fig. 1) is operated in the range from 13.3 to 1.33×10^6 newtons/meter^2. Provisions are incorporated to maintain the gas

source as an isothermal (304° K) source by using a temperature-monitored water supply circulating around the gas source container. A rotating piston and a Bourdon type gage calibrated by the National Bureau of Standards are attached to the gas source as a pressure standard. The operating range of the gas-source pressure, combined with the other controlled parameters of the system, provides a total variability adequate to perform calibrations from 6.65×10^{-11} to 1.33×10^{-5} newton/meter^2.

Porous plug.- A porous silicate glass network provides the pressure attenuation between the pressure source and the molecular furnace. The conductance of porous silicate glass is essentially constant over a large temperature range. Therefore, porous silicate glass makes a suitable material for this application, since high-temperature degassing is essential for proper system performance. Porous silicate glass has been fabricated (ref. 2) with a mean pore diameter of about 3×10^{-7} centimeter. The mean pore diameter of the porous plug is the determining factor for the maximum upstream pressures that can be used to insure molecular flow conditions. It is important to maintain molecular flow conditions throughout the system for maximum accuracy. Deviation from the molecular flow conditions becomes large when the mean free path (average distance that a molecule travels between successive collisions) decreases so that it is on the order of the magnitude of the mean pore diameter of the porous plug. The mean free path of argon (a typical calibration gas) at 304° K is given in reference 3 (p. 32) as 5.31×10^{-7} centimeter for a pressure of 1.33×10^6 newtons/meter^2. Since the maximum source pressure will not exceed 1.33×10^6 newtons/meter^2, molecular flow conditions will prevail throughout the porous plug. Since the source pressure is much greater than the furnace pressure, the gas flow rate through the porous plug is given by

$$Q_p = C_p p_t \tag{1}$$

where C_p is defined as the conductance of the porous plug and p_t is the gas-source pressure. The conductance of the porous plug is an experimentally determined parameter. If a gas source of known volume and pressure is attached to a porous plug and when steady-state flow conditions exist through the plug, molecular conservation requires that

$$-\frac{V\, dp_t}{dt} = C_p p_t \tag{2}$$

where V is the volume of a gas source container and t is the time in seconds. The solution to this equation gives

$$p_t = p_o \epsilon^{-C_p t/V} \tag{3}$$

If the initial pressure p_o, the pressure p_t, and the volume V are known, then the conductance can be determined from experimental data (fig. 2) over a desired pressure range.

A typical value for the conductance of the porous plug for argon at 304° K was determined to be on the order of 1.01×10^{-5} liter/second, with an experimental gas source volume of 1.35×10^{3} centimeters^3.

Molecular furnace.- The molecular furnace consists of a chamber in which a volume of gas is maintained at a constant and uniform temperature (same temperature as gas source) and a constant pressure. Uniform temperature requires that the gas is everywhere in thermal equilibrium with the wall of the molecular furnace. The furnace has a precision aperture with a value of δ_a/r_a of 2×10^{-3} from which a molecular beam is extracted. All aperture dimensions were chosen so that the flow is always in the free molecular regime. Therefore, for a given calibration gas the aperture conductance is constant, since the gas temperature (304° K) is maintained constant. An auxiliary cryopump maintains the pressure surrounding the outside of the molecular furnace several orders of magnitude below the furnace pressure. Consequently, the outside pressure may be neglected in calculating the flow rate. Under these conditions, the gas flow rate through the molecular furnace is given by

$$Q_a = C_a p_f \tag{4}$$

The temperature of the furnace is maintained at the temperature of the gas source. Therefore, under steady-state conditions, the flow rate in equations (1) and (4) must be equal. When these conditions exist, the furnace pressure is given as

$$p_f = \frac{C_p}{C_a} p_T \tag{5}$$

The values of C_p and C_a are chosen so that free molecular flow exists through the porous plug and molecular furnace. It was determined that typical values for C_p using argon at 304° K are on the order of 1.01×10^{-5} liter/second. Calculations from molecular theory show that C_a is 17.6 liters/second for an aperture radius r_a of 0.75 centimeter. The maximum and minimum values of source pressure were given as 1.33×10^{6} and 13.3 newtons/meter^2. Therefore, the maximum and minimum values of the furnace pressure p_f are 7.63×10^{-2} and 7.63×10^{-6} newton/meter^2, respectively.

MOLECULAR BEAM NUMBER DENSITY CALCULATIONS

Two methods are presented for analyzing the molecular beam number density. In the first method, the pressure is calculated on the axis of the beam. This method is restricted for use with a nude gage mounted 117 centimeters from the beam aperture. This restriction is used to insure that the solid angle subtended by the gage elements is sufficiently small so that density variations across the gage are negligible. In the second method, a temperature-monitored gage enclosure is mounted on the axis of the beam with

a known aperture conductance. With this method the molecular beam is transformed to an equilibrium gas upon entering the gage enclosure. Therefore, this method may be used for pressure calculations at 10 and 117 centimeters from the beam aperture.

Without Gage Enclosure

From the furnace aperture, a molecular beam is extracted (fig. 1) which radiates into the 2π half-space outside the furnace with an angular distribution according to the Knudsen Cosine Law (ref. 4). The beam is restrained to a small solid angle by cryopumping apertures as it enters the cryopumped cavity. The density of the beam is essentially uniform across its core due to the small solid angle it subtends. Since the beam temperature is only moderate and the cryopump temperature is low, the molecular capture probability at the cryopumped surface is near unity (ref. 5). Because of the high capture probability at the cryopumped surface, the molecular flux scattered in the forward direction by the collimating apertures and the backscattered flux from the far end of the cryopump produce a background pressure that is negligible. Therefore, background pressure may be neglected in comparison to the relatively high density of the beam core. The molecular beam core flux in a solid angle through a plane perpendicular to the beam axis is given in reference 4 (p. 11) as

$$\dot{N}_b = \frac{d\omega}{4\pi} A_a n_f \bar{v}_f \cos\theta \tag{6}$$

where $d\omega$ is a solid angle at an angle θ with the axis of the beam (aperture normal). For a detector of area A_d at a distance l along the normal to the beam aperture, the molecular flux is also given by

$$\dot{N}_b = n_b A_d \bar{v}_b \tag{7}$$

By comparing equations (6) and (7) and noting that the solid angle $d\omega$ subtended by a detector is A_d/l^2, it is found that the molecular beam density on the axis of the beam is

$$n_b = \frac{1}{4\pi} \frac{A_a}{l^2} \frac{\bar{v}_f}{\bar{v}_b} n_f \tag{8}$$

The average velocity in the furnace is related to that in the beam by

$$\bar{v}_f = \frac{8}{3\pi} \bar{v}_b \tag{9}$$

as shown in the appendix. Substitution of equation (9) into equation (8) gives the molecular beam density for a circular aperture of radius r_a as

$$n_b = \frac{2}{3\pi} \left(\frac{r_a}{l}\right)^2 n_f \tag{10}$$

The average molecular velocity of the beam is given by equation (9), which is greater by a factor of $3\frac{\pi}{8}$ than the average molecular velocity in the furnace. These molecular velocities are on the order of 10^4 centimeters/second as compared with velocities of the ionizing electrons which are greater than 10^6 centimeters/second for an ionization gage used for pressure measurements. Therefore, the beam molecules appear essentially stationary to the ionizing electrons of an ionization gage, and the effect of a molecular beam at moderate temperatures does not change the ionic generation from that produced in an equilibrium gas. It is known that an ionization gage gives an output proportional to a number density as given by equation (10) and cannot distinguish between a molecular beam or equilibrium gas at moderate temperatures. In view of these conditions, the effective beam pressure for ionization gage measurements can be related to the furnace pressure by substituting equations (5) and (9) into equation (8) to give the following calibration equation in the molecular beam:

$$p_b = \frac{2}{3\pi}\left(\frac{r_a}{l}\right)^2 \frac{C_p}{C_a} p_\tau \tag{11}$$

An assumption implicit in the derivation of the calibration equation is that the beam density is considered to be constant over the volume of a pressure indicating device such as an ionization gage. If the density variations perpendicular to and parallel to the beam axis are considered separately, equation (6) reveals that the density variation perpendicular to the beam axis is negligible as long as $\cos \theta/2$ is sufficiently close to unity. Beam density variations parallel to the beam axis may be investigated by applying equation (10) to different values of l. Letting l_{mid} and l_{ext} represent, respectively, the midpoint and one extreme of the ionization region of length L along the beam axis, the ratio of densities at these points is given by

$$\frac{n_{b,l_{ext}}}{n_{b,l_{mid}}} = \left(\frac{l_{mid}}{l_{mid} + \frac{L}{2}}\right)^2 \approx 1 - \frac{L}{l_{mid}} \tag{12}$$

for $l \gg L$. Therefore, to avoid a nonuniformity of gas density throughout an ionization gage volume, there is a minimum distance which must be maintained between the gage and beam aperture. To obtain a sufficiently large change in beam density with maximum accuracy over L by varying l, the calibration apparatus becomes long and cumbersome since the range of beam density variation is proportional to $\left(\frac{l_{maximum}}{l_{minimum}}\right)^2$. For example, if an error of less than 1/2 percent is required in equation (11), then for L = 4 centimeters l_{mid} must be $\geqq$800 centimeters. The error at l = 200 centimeters is 2 percent. The pressure in a molecular beam given by equation (11) was calculated from kinetic theory. In the next section, a different calibration equation is obtained when methods are employed to eliminate errors due to density variations parallel to the beam

axis. For this system, a value of $l_{maximum}$ of 117 centimeters was chosen, which corresponds to a variation of less than 4 percent. Smaller variations may be obtained by increasing l; however, it is necessary to restrict the physical size of the calibration apparatus for laboratory use. Therefore, a method was introduced where density variations could be small without excessive length.

With Gage Enclosure

To eliminate any error due to beam density variation parallel to the beam axis, an ionization gage is mounted in an isothermal enclosure so that the molecular beam is transformed to an equilibrium gas upon entering the enclosure. This principle can be used also to calibrate gages that would otherwise, due to their geometry, strongly scatter the molecular beam. It is shown by equation (A2) in the appendix that for a gas in equilibrium at temperature T, the number of molecules per unit area per second crossing from one side of an aperture with a given speed through a unit area making an angle ϕ with the normal to an aperture is

$$d\Gamma_v = nv \cos \phi \left(\frac{d\omega}{4\pi}\right) f(v)\, dv \tag{13}$$

For a small ideal molecular beam aperture (fig. 3) in the wall of a vessel (molecular furnace) containing a gas in equilibrium where the mean free path of the gas is large compared with the aperture diameter, the number of molecules per unit area and unit time passing through the aperture is given by equation (13). The molecules effuse from the beam aperture and enter the gage enclosure aperture. The molecular beam and gage enclosure apertures are nearly ideal since δ_a/r_a and δ_e/r_e are 2×10^{-3} and 5×10^{-3}, respectively. A nude ionization gage is mounted in the gage enclosure in a position such that it does not have a direct line of sight to the enclosure aperture. Since the mean free path of the calibration gas is long, there are a negligible number of molecular collisions in the space between A_1 and A_2; therefore, those molecules moving into a solid angle $d\omega$ from dA_1 pass through an elementary area dA_2 where

$$d\omega = \frac{\cos \phi\, dA_2}{Z^2} = \frac{l\, dA_2}{Z^3}$$

By substituting for $f(v)$ and $d\omega$, equation (13) becomes

$$d\Gamma_v = \frac{nl^2}{Z^4}\left(\frac{m}{2\pi kT}\right)^{3/2} dA_2 v^3 \epsilon^{-(m/2kT)^2 v^2} dv$$

thus, it follows that

$$d\Gamma = \frac{nl^2}{Z^4}\left(\frac{m}{2\pi kT}\right)^{3/2} dA_2 \int_0^\infty v^3 \epsilon^{-(m/2kT)^2 v^2} dv = \frac{nl^2}{4\pi}\left(\frac{8kT}{\pi m}\right)^{1/2} \frac{dA_2}{Z^4}$$

From the appendix the mean velocity in the furnace is given as

$$\bar{v}_f = \left(\frac{8kT_f}{\pi m}\right)^{1/2}$$

therefore,

$$d\Gamma = \frac{n_f \bar{v}_f}{4\pi} \frac{\ell^2 \, dA_2}{\left(\ell^2 + r^2 + r_1^2 - 2rr_1 \cos\theta\right)^2}$$

The total molecular flux into dA_2 from all the aperture area A_1 is then

$$d\dot{N}_{in} = \int_{A_1} d\Gamma \, dA_1 = \frac{n_f \bar{v}_f \ell^2 \, dA_2}{4\pi} \int_0^{r_a} \int_0^{2\pi} \frac{r \, dr \, d\theta}{\left(\ell^2 + r^2 + r_1^2 - 2rr_1 \cos\theta\right)^2}$$

Integrating with respect to θ gives

$$d\dot{N}_{in} = \frac{n_f \bar{v}_f \ell^2 \, dA_2}{4\pi} \int_0^{r_a} \frac{2\pi r\left(\ell^2 + r_1^2 + r^2\right) dr}{\left[\left(\ell^2 + r^2 + r_1^2\right)^2 - 4r_1^2 r^2\right]^{3/2}}$$

Integrating again with respect to r yields

$$d\dot{N}_{in} = \frac{n_f \bar{v}_f \, dA_2}{8} \left\{1 - \frac{r_1^2 + \ell^2 - r_a^2}{\left[\left(r_1^2 + \ell^2 + r_a^2\right)^2 - 4r_1^2 r_a^2\right]^{1/2}}\right\}$$

Now integrating the molecular flux into A_2 over all of aperture area A_2 gives the total number of molecules entering A_2 per unit time as

$$\dot{N}_{A2,in} = \int_{A_2} = \frac{n_f \bar{v}_f}{8} \int_0^{r_e} 2\pi \left\{1 - \frac{r_1^2 + \ell^2 - r_a^2}{\left[\left(r_1^2 + \ell^2 + r_a^2\right)^2 - 4r_1^2 r_a^2\right]^{1/2}}\right\} r_1 \, dr_1$$

Integrating this equation with respect to r_1 gives

$$\dot{N}_{A2,in} = \frac{\pi}{8} n_f \bar{v}_f \left(r_a^2 + r_e^2 + \ell^2\right) \left\{1 - \left[1 - \frac{4r_a^2 r_e^2}{\left(r_a^2 + r_e^2 + \ell^2\right)^2}\right]^{1/2}\right\} \tag{14}$$

In this application the enclosure radius is chosen so that ionic pumping does not introduce large errors in the gage enclosure pressure. This constraint means that the

conductance of the gage enclosure aperture C_e must be greater than 200 times the pumping speed of an ion gage (ref. 6, p. 89, and ref. 7, p. 93) to maintain an error less than 1/2 percent. Therefore, for a typical Bayard-Alpert gage structure r_e must be $\geqq$0.8 centimeter, and for a standard Redhead gage structure r_e must be $\geqq$3.1 centimeters.

The practical upper limit for the beam aperture radius (0.75 centimeter) has been shown to be set by the furnace pressure to maintain molecular flow conditions. To minimize scattering, the closest approach of the gage enclosure to the beam aperture is fixed at 10 centimeters to insure that gas effusion from the gage enclosure is negligible. By using the parameters r_a, r_e, and l, the quantity to the one-half power in equation (14) is approximately equal to 1 since

$$\frac{4r_a^2r_e^2}{\left(r_a^2 + r_e^2 + l^2\right)^2} \leqq \frac{1}{500}$$

Therefore, this expression may be expanded in a binomial series to give

$$\left[1 - \frac{4r_a^2r_e^2}{\left(r_a^2 + r_e^2 + l^2\right)^2}\right]^{1/2} \approx 1 - \frac{2r_a^2r_e^2}{\left(r_a^2 + r_e^2 + l^2\right)^2}$$

in which all but the first two terms are neglected. Equation (14) then becomes

$$\dot{N}_{A2,in} = \frac{n_f\bar{v}_f\pi}{4} \frac{r_a^2r_e^2}{r_a^2 + r_e^2 + l^2} \tag{15}$$

The beam molecules, after being randomized in the gage enclosure at T_e, then emerge from the enclosure aperture as a molecular beam. The number of molecules effusing per second is (ref. 8)

$$\dot{N}_{A2,out} = \frac{\pi}{4} n_e\bar{v}_e r_e^2 \tag{16}$$

In the process of operation, an ionization gage traps gas into the gage elements by an effective pumping speed S_g so that, when the molecular density in the gage enclosure reaches equilibrium,

$$\dot{N}_{A2,in} = \dot{N}_{A2,out} + S_g n_e \tag{17}$$

Substitution of equations (15) and (16) into equation (17) yields

$$\left(1 + \frac{4S_g}{\pi r_e^2\bar{v}_e}\right)n_e = \frac{\bar{v}_f}{\bar{v}_e} \frac{r_a^2}{r_a^2 + r_e^2 + l^2} n_f \tag{18}$$

A constraint has been imposed that the conductance of the enclosure aperture C_e must be greater than the pumping speed of an ionization gage S_g, that is,

$$\frac{\pi}{4}\bar{v}_e r_e^2 >> S_g$$

Equation (18) may therefore be written as

$$n_e = \frac{\bar{v}_f}{\bar{v}_e}\frac{r_a^2}{r_a^2 + r_e^2 + l^2} n_f \tag{19}$$

For a gas in equilibrium

$$\frac{\bar{v}_f}{\bar{v}_e} = \left(\frac{T_f}{T_e}\right)^{1/2}$$

and the molecular density in the furnace is related to that in the isothermal gas source by equation (5) to give

$$n_f = \frac{C_p n_\tau}{C_a}$$

The steady-state molecular density in the gage enclosure is therefore

$$n_e = \left(\frac{T_f}{T_e}\right)^{1/2}\frac{r_a^2}{r_a^2 + r_e^2 + l^2}\frac{C_p}{C_a} n_\tau$$

By using the ideal gas law this equation may be expressed in terms of pressure as

$$p_e = \left(\frac{T_e}{T_f}\right)^{1/2}\frac{r_a^2}{r_a^2 + r_e^2 + l^2}\frac{C_p}{C_a} p_\tau \tag{20}$$

For this treatment T_e and T_f are maintained at the same temperature. One can observe that the pressure is increased by an approximate factor of 5 when an enclosure is mounted about an ionization gage. Equation (20) must always be used when making calculations at l = 10 centimeters to account for density variations parallel to the beam axis. At l = 117 centimeters, the gage enclosure may or may not be used.

The various system calibration parameters are as follows:

C_a	17.6 liters/second
C_p	1.01×10^{-5} liter/second
l	10 to 117 centimeters

p_τ 13.3 to 1.33×10^6 newtons/meter^2

r_a 0.75 centimeter

r_e 0.8 to 3.1 centimeters

The theoretical calibration range is presented in figure 4. This pressure range is obtained by changing two parameters, the position l of a detector and the gas-source pressure p_τ.

PROBABLE ERROR IN PRESSURE MEASUREMENTS

Various parameter measurements are required to utilize equations (11) and (20) for pressure calculations. Each parameter has some uncertainty associated with its measurement. The numerical values of the following parameters are required for a pressure measurement:

(1) Pressure of source

(2) Temperature of molecular furnace and gage enclosure

(3) Radius of molecular beam aperture

(4) Radius of gage enclosure aperture

(5) Porous-plug conductance

(6) Distance from beam aperture to region of pressure measurement

To determine the magnitude of the probable error involved in a pressure measurement, it was necessary to evaluate the effects of uncertainties in each parameter measurement and this was done by choosing typical operating parameters and representative error data specifications for the pressure standard. For example, for $l = 10$ centimeters and with the gage enclosure, the pressure on the beam axis is given by equation (20) which is

$$p_e = \left(\frac{T_e}{T_f}\right)^{1/2} \frac{r_a^2}{r_a^2 + r_e^2 + l^2} \frac{C_p}{C_a} p_\tau$$

where

$$C_a = \frac{\pi}{4} \bar{v}_f r_a^2$$

and

$$C_p = \frac{V}{t} \ln \frac{p_o}{p_t}$$

Substituting for $\bar{v}_f$ from the appendix then gives

$$p_e = \left(\frac{2m}{\pi k}\right)^{1/2} \frac{(T_e)^{1/2}}{T_f} \frac{V}{t} \frac{p_\tau \ln \frac{p_o}{p_t}}{r_a^2 + r_e^2 + l^2}$$

The volume used for the porous-plug conductance measurement is determined by measuring the mass of a test fluid M (water) of a known density ρ required to fill the source volume, where $\rho = \frac{M}{V}$. Therefore, the equation for pressure may be written as

$$p_e = \left(\frac{2m}{\pi k}\right)^{1/2} \frac{(T_e)^{1/2}}{\rho T_f} \frac{M}{t} \frac{p_\tau \ln \frac{p_o}{p_t}}{r_a^2 + r_e^2 + l^2} \tag{21}$$

Calculations of the fractional error in a measurement are obtained by knowing the uncertainty in individual parameters. The fractional error in p_e is (ref. 9, pp. 2-9)

$$\frac{\Delta p_e}{p_e} = \pm \sum_{j=1}^{10} \left| \frac{1}{p_e} \frac{\partial p_e}{\partial x_j} \Delta x_j \right| \tag{22}$$

where Δx_j is a measurement uncertainty in the x_j parameter. The values of m, π, ρ, and k are considered to be known with sufficient accuracy so that they are essentially constant for this treatment. Each partial differential factor and the measurement uncertainty for all parameters are listed in table 1. The values used for measurement uncertainty represent typical measurement errors and are based on experimentally acquired data. By using the results of table 1, equation (22) becomes

$$\frac{\Delta p_e}{p_e} = \pm \left[\left| \frac{2r_a \Delta r_a}{r_a^2 + r_e^2 + l^2} \right| + \left| \frac{2r_e \Delta r_e}{r_a^2 + r_e^2 + l^2} \right| + \left| \frac{2l \Delta l}{r_a^2 + r_e^2 + l^2} \right| + \left| \frac{\Delta p_o}{p_t \ln \frac{p_o}{p_t}} \right| + \left| \frac{\Delta p_t}{p_t \ln \frac{p_o}{p_t}} \right| \right.$$

$$\left. + \left| \frac{\Delta p_\tau}{p_\tau} \right| + \left| \frac{\Delta M}{M} \right| + \left| \frac{\Delta T_e}{2T_e} \right| + \left| \frac{\Delta T_f}{T_f} \right| + \left| \frac{\Delta t}{t} \right| \right]$$

By substituting for each parameter and measurement uncertainty, the fractional error in p_e is found to be 5.97 percent. Similar calculations were made for l = 117 centimeters, and the fractional error in p_e was found to be 5.99 percent. For pressure measurements without the gage enclosure at l = 117 centimeters, the fractional error in p_b is 5.82 percent. The percent error quoted here represents a maximum since it was

assumed that the distribution of all errors would add. This procedure was used in order to reflect the largest magnitude of percent error. Therefore, a certain degree of confidence is placed in these values on the basis of the reliability of the uncertainty measurements. For any combination of measurement parameters, the greatest contribution in the fractional error for p_e and p_b is due to the l measurement. The larger contribution of the l measurement is due to the relative magnitude of l with respect to the other parameters.

EXPERIMENTAL STUDY

A feasibility study was made for the Langley Research Center under contract NAS 1-5347 by Norton Research Corporation of Cambridge, Massachusetts, to examine the practical applications of a molecular beam system. The pertinent results obtained from a comparison of ionization gage pressure indications and number density calculations from molecular theory are presented in this section. For this study a prototype calibration system was installed in an extreme-high vacuum chamber to make use of the low-temperature helium gas cryopumped cavity for attaining low-density background conditions. A cylindrical cryopumped cavity (fig. 5) 45 centimeters in diameter and 60 centimeters long was used for this experiment. The gas source (argon), molecular furnace, and gage enclosure were maintained isothermal by a common heat-transfer line circulating fluid around them. Water was used as the heat-transfer fluid for which the temperature was maintained at 304° K.

A nude ionization gage was utilized in the enclosure for pressure monitoring. The gage was operated at an emission current of 3×10^{-3} ampere which corresponded to a sensitivity of 9.02×10^{-4} ampere/(newton/meter^2) for argon. The nude ionization gage was calibrated prior to and after the experimental beam study. No significant change was noted in the gage sensitivities. Since uncertainties always exist for ionization gage calibrations, a conservative value of 8 percent is assumed for the pressure indications of the calibrated gage. This uncertainty is believed to be typical and representative of uncertainties associated with standard gage calibrations. The orifice on the enclosure was a 1.6-centimeter-diameter hole in a 40-micrometer-thick, certified oxygen free, high-conductivity copper foil. The conductance of this orifice was sufficiently large to make ionic pumping by the gage negligible; this imposed the constraint that C_e be much greater than S_g. Bakeout of the enclosure was accomplished by electron bombardment from the gage filament during which time the water cooling was diverted to flow around the molecular furnace and gas source only. Bakeout of the gage and enclosure was carried out at 622° K for several hours until activation of the electron bombardment at 1.33×10^{-7} newton/meter^2 caused no increase in the collector ionization current. Ultrapure argon was admitted to the calibration system, in which was utilized a porous plug

with a conductance of 6.84×10^{-6} liter/second and a beam orifice diameter of 0.38 centimeter for a gage at $l = 50$ centimeters. Fabricating techniques for porous plugs are such that the same conductance cannot be precisely duplicated for the same material and porous-plug size. Therefore, the conductance value quoted herein for the experimental study is different from that used in the construction of the final system. Table 2 gives the gage pressure readings and calculated pressures as determined from equation (20). The gage pressure readings were noted to be higher than the calculated pressures, but not greater than 10 percent. This advanced technique is capable of producing molecular number densities for gage calibrations down to 1.33×10^{-11} newton/meter^2 which is beyond the X-ray limit of most hot-filament ionization gages. No suitable data could be obtained from the gage below 2.73×10^{-8} newton/meter^2 due to this X-ray limit. Corrections for X-ray currents were made on the gage indicated pressures shown in table 2 for the lower pressure ranges.

This preliminary work has shown that the beam system is experimentally feasible. Comparison of the calculated beam density with an ionization gage measurement indicated that the results were within the uncertainty associated with gage calibration and molecular beam system parameters.

CONCLUDING REMARKS

An analysis of an ultra-high vacuum molecular beam system utilizing a known density source has been presented. Limitations in the application of this system were shown to be dependent upon uncertainties associated with parameter measurements. Evaluation of the fractional error in the calculated beam pressures indicated that an uncertainty of 6 percent is attainable.

Experimental results obtained from a prototype molecular beam system support the feasibility of using this technique to generate a known molecular beam density. Calculated molecular beam densities were compared with ionization gage measurements in the pressure range from 10^{-8} to 10^{-5} newton/meter^2. Agreement was within 10 percent.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 9, 1969,
124-09-19-03-23.

APPENDIX

DERIVATION OF AVERAGE MOLECULAR SPEED IN A MOLECULAR BEAM

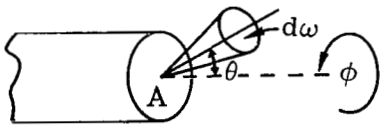
Maxwell's distribution law (ref. 10) for an equilibrium gas at temperature T gives the fraction of molecules with speeds in the range dv to be

$$f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \epsilon^{-\beta^2 v^2} dv \tag{A1}$$

where $\beta^2 = \frac{m}{2kT}$. Calculation of the average molecular speed yields

$$\bar{v} = \frac{\int_0^\infty vf(v)\,dv}{\int_0^\infty f(v)\,dv} = \frac{\int_0^\infty v^3 \epsilon^{-\beta^2 v^2}}{\int_0^\infty v^2 \epsilon^{-\beta^2 v^2} dv} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

Consider the following sketch which illustrates molecular effusion:



This configuration consists of a Maxwellian gas of volume V with molecules moving equally in all directions so that a fraction of molecules $d\omega/4\pi$ moves into the solid angle $d\omega$ whose axis is inclined at an angle θ to the normal of A. The molecules that are included in a cylinder with base A and slant height $v\,dt$ will cross A in a finite time dt. Therefore, the number of molecules crossing A per unit area per unit time with velocities in a given range dv and passing into $d\omega$ at θ to the normal is given as

$$d\Gamma_v = nv \cos\theta \frac{d\omega}{4\pi} f(v)\,dv \tag{A2}$$

By using spherical coordinates, the elementary solid angle $d\omega$ may be expressed as

$$d\omega = \sin\theta\, d\theta\, d\phi$$

and

$$d\Gamma_v = nAv^3 \epsilon^{-\beta^2 v^2} dv \sin\theta \cos\theta\, d\theta\, d\phi$$

Since v^3 appears in this expression, rather than v^2 as in equation (A1), it is suspected that of the gas molecules passing through a plane from one side in a given time,

the faster molecules pass through more frequently. This result may be expected since faster molecules traverse more of the volume in a given time and, thus, have a greater probability of being observed at plane A. The average molecular speed may be calculated for those molecules passing through the area A into the half-space beyond. Therefore,

$$\bar{v}_b = \frac{\int_0^\infty v \, d\Gamma_v}{\int_0^\infty d\Gamma_v} = \frac{\int_0^{2\pi} \int_0^{\pi/2} \int_0^\infty v^4 \epsilon^{-\beta^2 v^2} dv \sin\theta \cos\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi/2} \int_0^\infty v^3 \epsilon^{-\beta^2 v^2} dv \sin\theta \cos\theta \, d\theta \, d\phi}$$

Evaluating this integral gives

$$\bar{v}_b = \frac{3\pi}{8}\left(\frac{8kT}{\pi m}\right)^{1/2}$$

Recalling that

$$\bar{v}_f = \left(\frac{8kT_f}{\pi m}\right)^{1/2}$$

therefore,

$$\bar{v}_b = \frac{3}{8}\pi \bar{v}_f$$

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TABLE 1.- MEASUREMENT UNCERTAINTY DATA

Parameter	Differential factor	Measurement uncertainty
x_j	$\frac{1}{p_e}\frac{\partial p_e}{\partial x_j}$	Δx_j
l = 10 centimeters	$\frac{-2l}{r_a^2 + r_e^2 + l^2}$	0.25 centimeter
M = 1350 grams	$\frac{1}{M}$	6.75 grams
$p_o = 241.15 \frac{\text{newtons}}{\text{meter}^2}$	$\frac{1}{p_o \ln \frac{p_o}{p_t}}$	$0.36 \frac{\text{newton}}{\text{meter}^2}$
$p_t = 6.89 \frac{\text{newtons}}{\text{meter}^2}$	$\frac{-1}{p_t \ln \frac{p_o}{p_t}}$	$0.01 \frac{\text{newton}}{\text{meter}^2}$
$p_\tau = 13.3 \frac{\text{newtons}}{\text{meter}^2}$	$\frac{1}{p_\tau}$	$0.02 \frac{\text{newton}}{\text{meter}^2}$
r_a = 0.75 centimeter	$\frac{-2r_a}{r_a^2 + r_e^2 + l^2}$	0.01 centimeter
r_e = 0.8 centimeter	$\frac{-2r_e}{r_a^2 + r_e^2 + l^2}$	0.01 centimeter
$T_e = 304^o$ K	$\frac{-1}{2T_e}$	0.51^o K
$T_f = 304^o$ K	$\frac{-1}{T_f}$	0.51^o K
$t = 4.75 \times 10^5$ seconds	$\frac{-1}{t}$	47.52 seconds

TABLE 2.- ARGON BEAM DATA

Gas source pressure, newtons/meter^2	Gage pressure, newtons/meter^2	Calculated pressure, newtons/meter^2
2.87×10^2	2.73×10^{-8}	2.54×10^{-8}
1.04×10^2	9.89×10^{-7}	9.21×10^{-7}
7.75×10^3	7.23×10^{-7}	6.84×10^{-7}
2.88×10^3	2.65×10^{-7}	2.54×10^{-7}
9.44×10^4	8.80×10^{-6}	8.34×10^{-6}
5.16×10^4	4.82×10^{-6}	4.56×10^{-6}
1.38×10^4	1.29×10^{-6}	1.22×10^{-6}
9.38×10^5	9.23×10^{-5}	8.32×10^{-5}
4.79×10^5	4.58×10^{-5}	4.25×10^{-5}
2.58×10^5	2.52×10^{-5}	2.29×10^{-5}

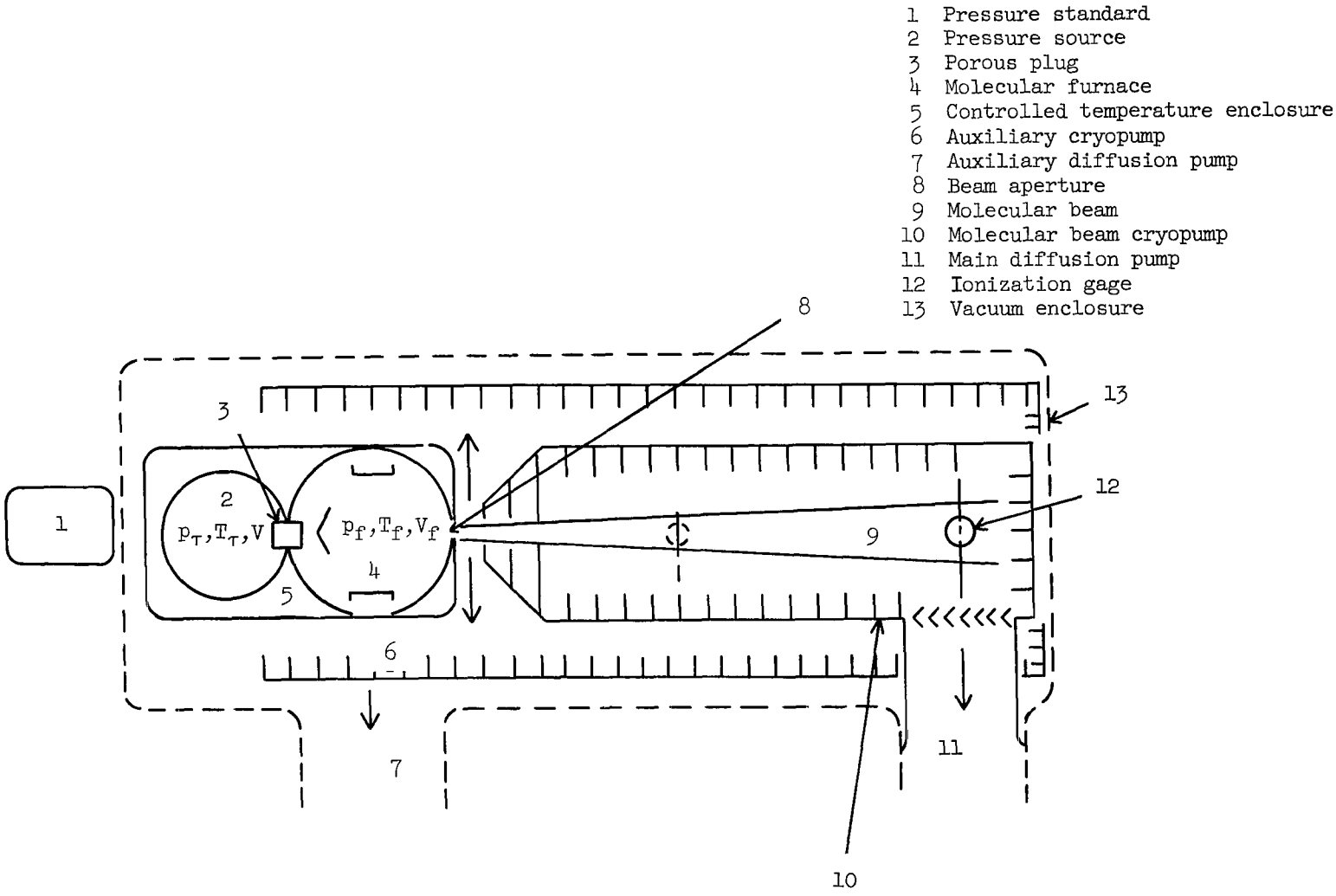


Figure 1.- Molecular beam calibration apparatus.

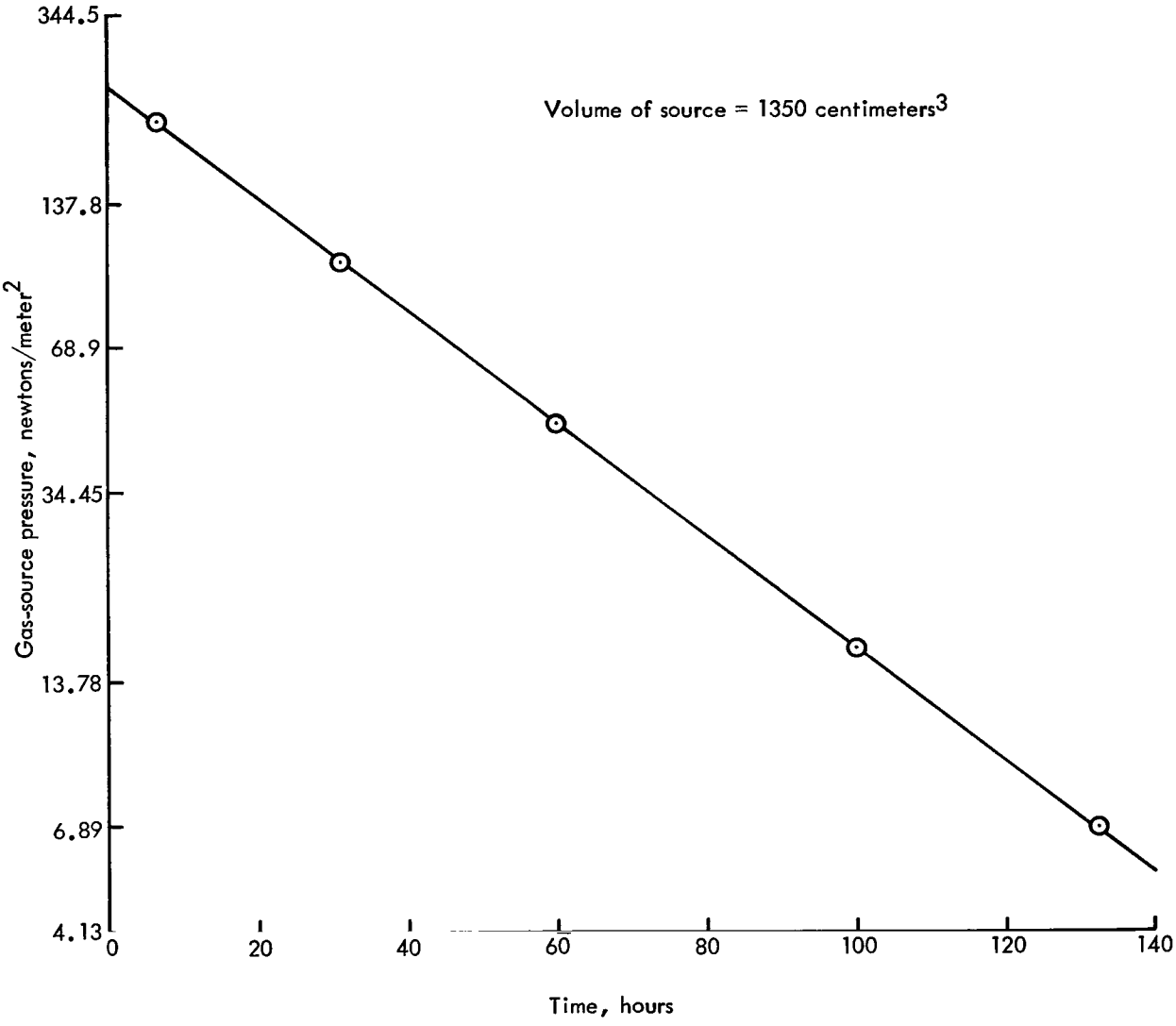
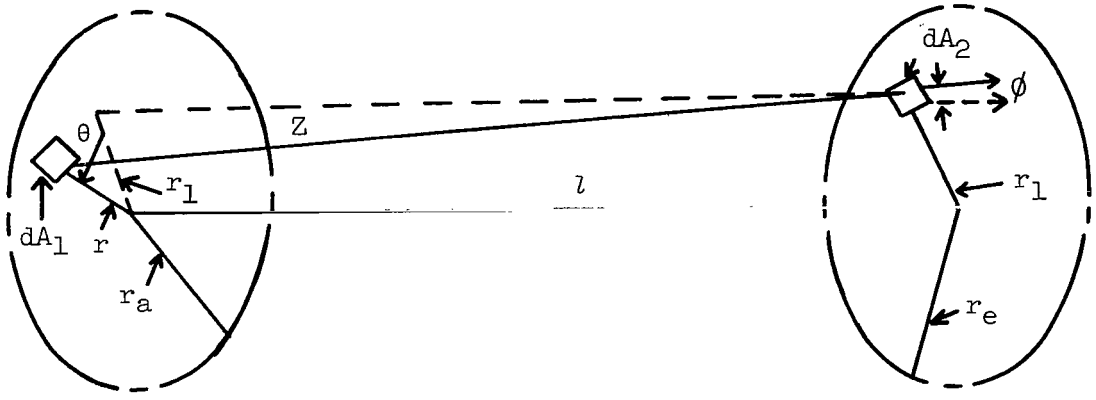
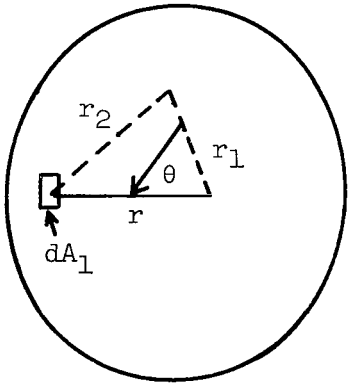


Figure 2.- Porous-plug conductance measurement.



Molecular beam aperture

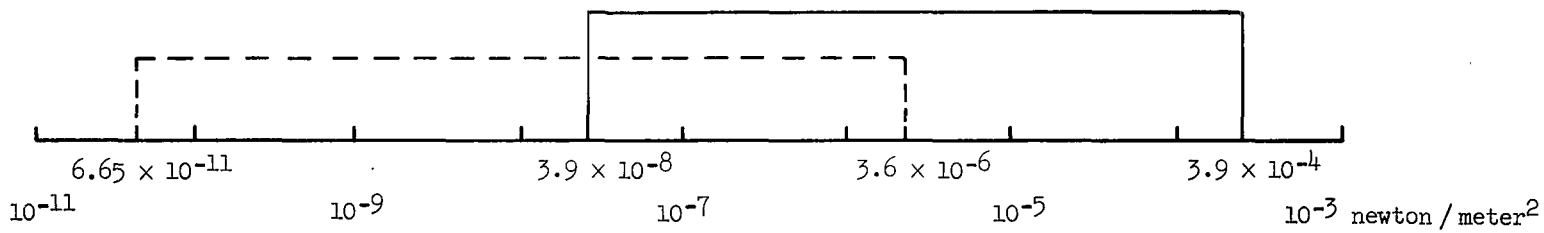
Gage enclosure aperture



$$Z^2 = l^2 + r_2^2$$

$$r_2^2 = r^2 + r_1^2 - 2rr_1 \cos \theta$$

Figure 3.- Molecular beam and gage enclosure apertures.



$$p_b = \frac{2}{3\pi}\left(\frac{r_a}{l}\right)^2 \frac{C_p}{C_a}\, p_T \text{ ---- Gage enclosure removed}$$

$$p_e = \left(\frac{T_e}{T_f}\right)^{1/2} \frac{r_a^2}{r_a^2 + r_e^2 + l^2} \frac{C_p}{C_a}\, p_T \text{ ——— Gage enclosure on}$$

Figure 4.- Pressure calibration range.

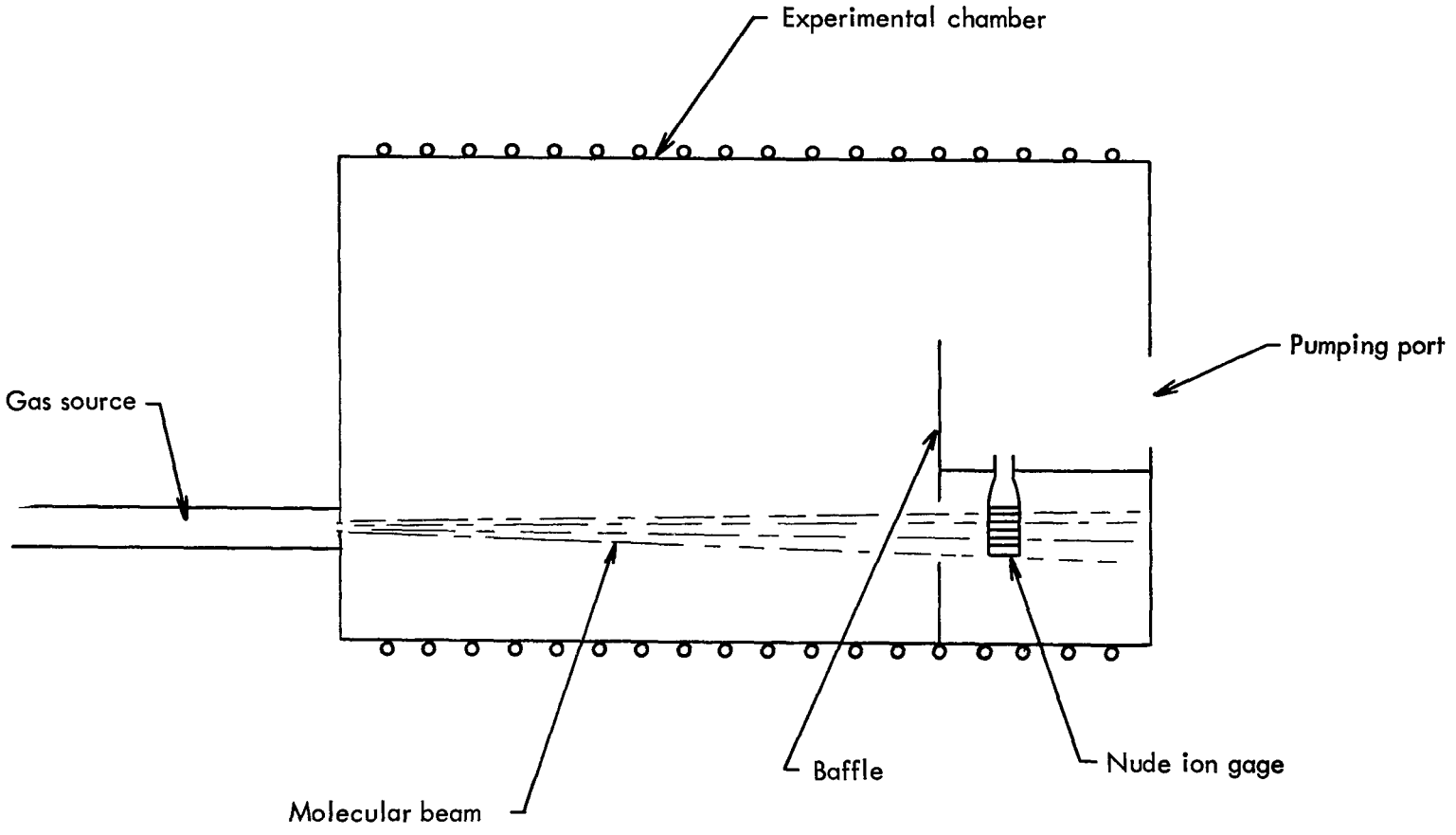


Figure 5.- Apparatus for argon beam experiment.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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